EEB 133
QUIZ 1 Answers

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The solutions to this homework are in *Mathematica*.

1. The population of Israel is about 6 million people, 80% of which are Jewish and 20% of which are Palestinian. The growth rate of the Jewish population is about 1.2% per year, and that of the Palestinians is about 3.5% per year. Assuming continued exponential growth, when do you anticipate the two will be equally frequent in Israel?

ANSWER: Let the current population of all of Israel be \( n_{i5} \), the population of Jews be \( n_{j5} \), and the population of Palestinians be \( n_{p5} \). The corresponding numbers at equality will be \( n_{i5} \), \( n_{j5} \) and \( n_{p5} \).

We know that:

\[
\begin{align*}
\text{In}[1]: & \quad n_{i5} = 6000000 \\
\text{Out}[1] & = 6000000 \\
\text{In}[2]: & \quad n_{j5} = 0.8 n_{i5} \\
\text{Out}[2] & = 4.8 \times 10^6 \\
\text{In}[3]: & \quad n_{p5} = 0.2 n_{i5} \\
\text{Out}[3] & = 1.2 \times 10^6 \\
\end{align*}
\]

We assume that growth is exponential. Let the rate of growth for Jews be \( r_{j} \) and that for Palestinians be \( r_{p} \).

\[
\begin{align*}
\text{In}[7]: & \quad r_{j} = 0.012 \\
\text{Out}[7] & = 0.012 \\
\text{In}[8]: & \quad r_{p} = 0.035 \\
\text{Out}[8] & = 0.035 \\
\end{align*}
\]

We want to find the time when the two are equal. This can be done with the `solve` command.

\[
\begin{align*}
\text{In}[9]: & \quad \text{Solve}[n_{j5} \times \text{Exp}[r_{j} \times t] = n_{p5} \times \text{Exp}[r_{p} \times t], \ t] \\
\text{Out}[9] & = \{ \{ t \rightarrow 60.2737 \} \}
\end{align*}
\]
So Palestinians and Jews should be equal in numbers by approximately 2065, and then the Palestinians should outnumber Jews thereafter, assuming current demography.

2. What is logistic growth? Describe the differential equation for it, and justify that equation. Give a graphic example of it.

ANSWER: Logistic population growth is defined for both continuous time and discrete generation models. The critical difference between exponential and logistic models is that the growth rate is presumed to be constant for exponential growth, but decrease linearly with population size for the logistic growth. The main justification for this is that as the population size increases, resources for each individual will decrease, resulting in a lowered rate of increase. For continuous time, the equation for logistic growth is \( \frac{dN}{dt} = r N(t) \frac{(K - N(t))/K}{K} \), where \( r \) and \( K \) are constants. For discrete time the logistic equation is \( \Delta N = r N(t) \frac{K - N(t)}{K} \).

Because the discrete logistic has some unpleasant stability properties, I will graph only the continuous case. The solution to the continuous case is known, and is:

```
In[17]:= n[n0_, r_, K_, t_] := K / (1 + (K / n0 - 1) Exp[-r t])
```

```
In[24]:= Plot[n[2, 0.1, 100, t], {t, 0, 100}, AxesLabel -> {time, population size} ]
```

```
Out[24]= - Graphics -
```

3. (grads only) How did Roughgarden fit parameters to the growth of Paramecium? Does this seem reasonable to you in view of the dependence of this method on sample size?

ANSWER: Roughgarden searched through the values of \( r \) and \( K \) until she found the values that minimized the chi-square fit between the model and the observation.

This is sensitive to the size of the observed populations, but because all values of \( r \) and \( K \) that were explored had the same observed population size, this should not matter.